

2) Scalar multiplication

Let $A = [a_{ij}]$ be $m \times n$ matrix and r is a real number then the scalar multiple of A by r

(denoted by rA) defined to the matrix $D = [d_{ij}]$, where $d_{ij} = r a_{ij}$, $1 \leq i \leq m, 1 \leq j \leq n$

Example $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $r = 2$, $2A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

3) Matrix multiplication

Let $A = [a_{ij}]$ be $m \times p$ matrix, $B = [b_{ij}]$ be $p \times n$ matrix then the product of A and B (denoted by

AB) defined to be the $m \times n$ matrix $C = [c_{ij}]$, where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{ip}b_{pj}$, $1 \leq i \leq m, 1 \leq j \leq n$

Example $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $AB = \begin{bmatrix} 13 & 18 & 23 \\ 16 & 22 & 28 \end{bmatrix}$

Let $A = [a_{ij}]$ be $m \times p$ matrix, $B = [b_{ij}]$ be $p \times n$ matrix

- 1) If $n \neq m$ then BA may not be defined

- 2) If $n = m$ then BA is $p \times p$, while AB is $m \times m$. Thus if $p \neq m$, AB and BA are different

sizes.

- 3) AB and BA are both of the same size. But $AB \neq BA$.

Example $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $AB = \begin{bmatrix} 13 & 18 & 23 \\ 16 & 22 & 28 \end{bmatrix}$, BA is not defined

Example $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $AB = \begin{bmatrix} 7 & 10 \\ 10 & 13 \end{bmatrix}$, $BA = \begin{bmatrix} 7 & 10 \\ 10 & 13 \end{bmatrix}$

$AB = BA$.

Example $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $AB = \begin{bmatrix} 7 & 10 \\ 10 & 13 \end{bmatrix}$, $BA = \begin{bmatrix} 7 & 10 \\ 10 & 13 \end{bmatrix}$

Properties of matrix addition

$$1) A+B = B + A$$

$$2) A+(B+C) = (A+B)+C$$

$$3) A+0 = 0+A$$

$$4) A+(-A) = (-A)+A = 0$$

Example $A =$, $B =$, $C =$,

$$A+B == B + A$$

$$(A+B)+C = + =$$

$$A+(B+C) = + =$$

$$-A =$$

$$A+(-A) = + =$$

$$(-A)+A = + =$$

$$A+0 = + =$$

$$0+A = +=$$

$$A-B = A +(-B)$$

Example $A =$, $B =$

$-B =$

$A - B = A + (-B) =$